

Homework 4, Due Thurs. Feb. 14

1) A two-dimensional velocity field is given as

$$u = x^2y \quad v = -xy^2$$

a) Write the equation of the family of streamlines.

b) Write the equation of the streamline that passes through the point (1,2).

2) Consider the velocity field

$$\vec{V} = (x^2 + y^2)\hat{i} + 2xy^2\hat{j}$$

a) Is the flow rotational or irrotational?

b) Calculate the circulation around the right triangle formed by connecting the points (0,0), (1,1), (1,0), using $\oint \vec{v} \cdot d\vec{s}$.

c) Calculate the circulation around the right triangle formed by connecting the points (0,0), (1,1), (1,0), using $\int \int (\nabla \times \vec{V}) \cdot \hat{n} dS$.

3) Is the flow field $v_r = 2r \sin \theta \cos \theta, v_\theta = -2r \sin^2 \theta$ irrotational?

4) The stream function of a two-dimensional incompressible flow is given by the equation

$$\psi = x^2 + 2y.$$

What is the magnitude and direction of the velocity at the point (2,3)?

5) Given the potential function $\phi = x^2 + x - y^2$ find the velocity field and the associated stream function. Using Maple, show the contour plot of both ϕ and ψ simultaneously. (Hint: to get two plots on the same graph use `contourplot({f,g},x=#..#,y=#..#)`.)

6) Given the potential function $\phi = r \cos \theta + \frac{225}{r} \cos \theta$.

a) Find the components of velocity v_r and v_θ .

b) Find the stream function as a function of r and θ .

c) Write the stream function in cartesian coordinates.

d) Using Maple, show the contour plot of the stream function. You will need to use a range on the order of -50 to 50.