# THE PRESSURE FIELD OF A GUST INTERACTING WITH A FLAT PLATE

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A semianalytical solution is presented for the unsteady pressure field of a vortical gust interacting with a flat-plate airfoil in subsonic flow. The solution will serve as a benchmark for evaluating the accuracy and efficiency of time dependent numerical schemes. The specific case considered corresponds to the ICASE benchmark problem number 6. The results are compared with those of asymptotic theories for high frequency and show excellent agreement.

## INTRODUCTION

The treatment of a two-dimensional gust impinging on a flat plate airfoil at subsonic speed is a classical problem in unsteady aerodynamics. The assumption of a mean uniform flow uncouples the unsteady flow problem from the mean flow and leads to the linearized Euler equations with constant coefficients. The physical problem depends on two parameters, the reduced frequency $k_1$ which is a measure of the convective time scale to the gust period, and the Mach number $M$ which is the ratio of the mean flow velocity $U_\infty$ to the speed of sound $a_\infty$. Although, the mathematical problem may appear to be relatively simple, *no exact* solution exists for the general case. In the early treatments, the problem was often formulated in terms of Possio's integral equation and solutions were obtained by collocation techniques [1, 2]. More recently, frequency-domain finite-difference solutions were obtained by Scott and Atassi [3]. Because of the widespread applications of unsteady airfoil theory to flutter and forced vibrations, asymptotic solutions were derived for the unsteady pressure jump along the plate surface for the low frequency [4, 5] and the high frequency cases [6, 7]. For more details, the reader is referred to a recent review article by Atassi [8].

Interest in the far-field acoustic radiation has motivated the development of methods to calculate the unsteady pressure field. Amiet [9] gave an expression for the far-field acoustic power produced by an airfoil in subsonic turbulent flow. However, he considered *only* the dipole contribution to the far-field sound. Martinez and Widnall [10] gave an exact expression for the far-field acoustic pressure in the limit of high frequency. Atassi et al. [11] derived an expression for the unsteady pressure everywhere in terms of the unsteady pressure jump along the plate surface for arbitrary values of the parameters $k_1$ and $M$. They calculated the unsteady pressure jump along the plate by solving Possio's integral equation. Their results show that as the frequency parameter $K_1 = \omega c/(2a_\infty \beta^2)$ becomes larger than $\pi/2$, quadrupole and noncompact source effects become *significant*. Here, $\omega$ is the circular frequency, $c$ is the plate chord length and $\beta^2 = 1 - M^2$.

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In the present paper, we use the method of Atassi et al. [11] to calculate the unsteady pressure resulting from a gust interacting with a flat-plate airfoil. The specific case considered corresponds to ICASE problem 6 for which the Mach number is 0.5 and the sinusoidal transverse gust has a reduced frequency $k_1 = \omega c/(2U_\infty) = 15\pi/4$. Since our solution relies on a Possio solver and thus is semianalytical, the results for both the unsteady pressure jump along the plate surface and the acoustic pressure in the far-field are compared with high frequency asymptotic theories [7, 10].

## MATHEMATICAL FORMULATION

Details of the mathematical derivation are given in [11]. The results can be summarized as follows. For an inviscid, non-heat conducting uniform mean flow, with an imposed upstream vortical disturbance, the linearized unsteady velocity field can be split into a convected vortical part and a potential part. Since the problem is linear, without loss of generality, we may consider a single Fourier component for the vortical part. Therefore, the velocity can be written as

$$\vec{V}(\vec{x},t) = U_\infty \vec{i}_1 + \vec{a} e^{i(\omega t - \vec{k}\cdot\vec{x})} + \vec{\bigtriangledown}\phi(\vec{x},t) \tag{1}$$

where $\vec{a} = (a_1, a_2, a_3)$ is the amplitude vector of the vortical disturbance, $\vec{k} = (k_1, k_2, k_3)$ is its wave number vector, and $\vec{\bigtriangledown}\phi$ is the potential part of the unsteady velocity. It is customary to normalize lengths with respect to half the chord, $c/2$, and velocities with respect to $U_\infty$. The unsteady pressure $p'$ is given by $p' = -\rho_\infty D_0\phi/Dt$. where $D_0/Dt \equiv \partial/\partial t + U_\infty \partial/\partial x_1$. The unsteady pressure $p'$ is governed by the convective wave equation

$$\frac{1}{a_\infty^2}\frac{D_0^2}{Dt^2}p' - \nabla^2 p' = 0 \tag{2}$$

and a similar equation can be derived for $\phi$. By introducing

$$P = \frac{p'}{\rho_0 a_2 U_\infty} e^{-i(k_1 t + MK_1\tilde{x}_1 - k_3\tilde{x}_3/\beta)} \tag{3}$$

equation (2) reduces to the two-dimensional Helmholtz equation in the Prandtl-Glauert coordinate system

$$\left(\tilde{\bigtriangledown}^2 + K^2\right) P = 0 \tag{4}$$

where the Prandtl-Glauert coordinates are $\tilde{x}_1 = x_1$, $\tilde{x}_2 = \beta x_2$, $\tilde{x}_3 = \beta x_3$, with $K_1 = k_1 M/\beta^2$, and $K^2 = K_1^2 - k_3^2/\beta^2$. Traditionally the boundary value problem for $P$ has been formulated in terms of a singular integral equation [1]. This equation is solved by direct collocation and gives the unsteady pressure along the plate, $\Delta p'$.

The unsteady pressure field is then obtained using Green's theorem [11],

$$p'(\vec{\tilde{x}}) = \frac{-i}{4} K\tilde{x}_2 \int_{-1}^{1} \Delta p'(\tilde{y}_1) e^{iMk_1(\tilde{x}_1-\tilde{y}_1)} \frac{H_1^{(2)}(K|\vec{\tilde{x}} - \vec{\tilde{y}}|)}{|\vec{\tilde{x}} - \vec{\tilde{y}}|} d\tilde{y}_1 \tag{5}$$

where $\Delta p' = p'(y_1, 0+) - p'(y_1, 0-)$ is the pressure jump along the plate surface, and $H_1^{(2)}$ is the Hankle function. This expression gives the unsteady pressure field everywhere in the plane. It accounts for both dipole and quadrupole effects. For large distance ($r = |\vec{x}| \to \infty$) this expression can be simplified and the unsteady pressure can be cast in terms of the Fourier transform of $\Delta p'$ [11].

$$p'(r, \theta) = \frac{\beta K^{1/2}}{\sqrt{8\pi}} \frac{e^{i(\pi/4)}}{\sqrt{r}} \frac{sin\theta}{(1 - M^2 sin^2\theta)^{3/4}} exp\{-ir[K(1 - M^2 sin^2\theta)^{1/2} - MK_1 cos\theta]\}$$

$$X \int_{-1}^{1} \Delta p'(y_1) e^{i\alpha y_1} dy_1 \tag{6}$$

where

$$\alpha = K \frac{cos\theta}{\sqrt{1 - M^2 sin^2\theta}} - MK_1$$

## ICASE BENCHMARK PROBLEM 6

In this case we have a transverse gust defined as

$$v = 0.1 a_\infty sin\left[\frac{\pi}{8}\left(\frac{x}{M_\infty} - t\right)\right] \tag{7}$$

where the normalization for the velocity is with respect to $a_\infty$; length, with respect to $\Delta x = 1$; and time, with respect to $\triangle x / a_\infty$. The Mach number is given as 0.5, and the chord is 30 units. Using the usual normalization, we get $k_1 = 15\pi/4 = 11.781$, and $a_2 = 0.2$. This corresponds to $K_1 = 7.85$, a high frequency case.

The unsteady pressure is to be calculated on a box surrounding the flat plate as shown in figure (1). The sides of the box are located at dimensional positions of $x = \pm 95$ and $y = \pm 95$. When nondimensionalized by the semichord the values are $x = \pm 6.333$, $y = \pm 6.333$. Thus, the box boundaries are not located in the far field and as a result, (5) must be used instead of its far-field expansion.

The fact that $K_1$ is relatively large allows us to compare our results for both $\Delta p'$ and the far-field acoustic pressure with the high frequency asymptotic theories [7, 10]. The results shown are for the normalized pressure

$$p = \frac{p'}{\rho_\infty a_\infty^2} \tag{8}$$

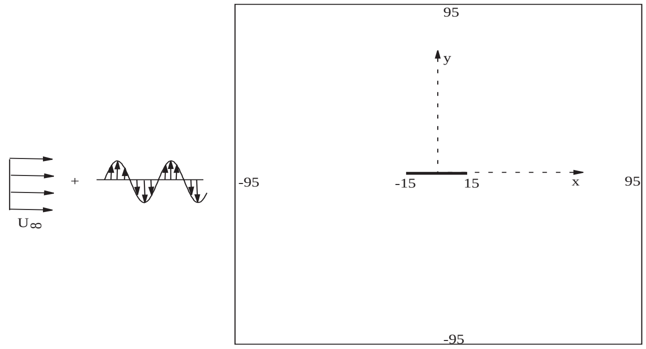


Figure 1: ICASE benchmark problem 6

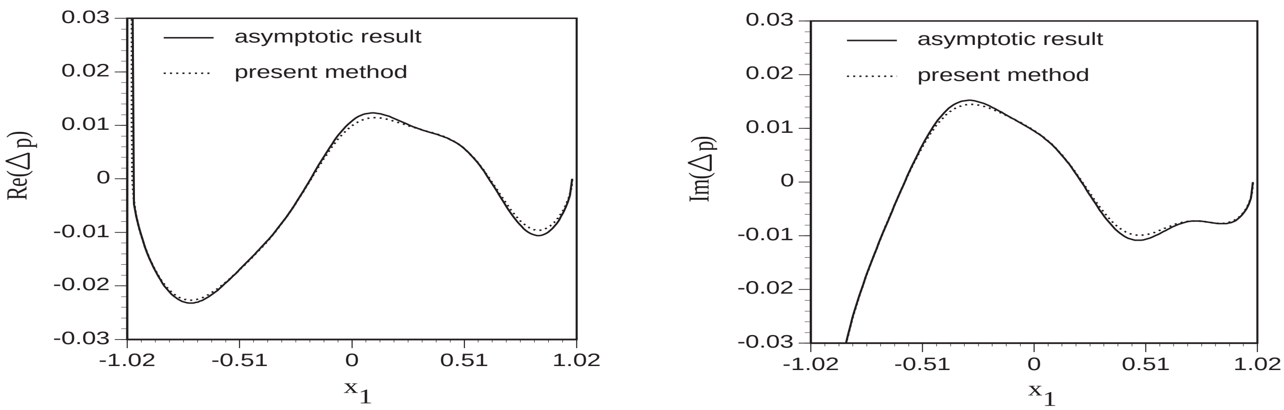


Figure 2: Unsteady pressure jump across airfoil

Figure 2 shows plots of the real and imaginary parts of the pressure jump $\Delta p$ using the present method and the asymptotic expression derived by Amiet [7] and Martinez and Widnall [10]. The excellent agreement shows the high accuracy of our results.

In order to compare our acoustic pressure with the far-field asymptotic expression of Martinez and Widnall [10], we used the far field expansion of (5). Figure 3 shows a comparison between directivity plots of $|p|\sqrt{r}$, using the two methods. Again the agreement is excellent.

The mean square pressure, $\overline{p^2}$ is now calculated at the ICASE box boundaries. Figure 4 shows the variation of $\overline{p^2}$ on the top boundary of the box. Because the pressure is antisymmetric with respect to the y axis, the values at the bottom boundary are the same as on the top boundary. Figure 5 shows the variation of $\overline{p^2}$ along the left and right boundaries of the box, respectively.

The authors would like to point out that the data for $\overline{p^2}$ presented a the workshop were calculated for a gust of amplitude $0.1U_\infty$, while the gust amplitude of the present data is $0.1a_\infty$ (see equation (7)). Therefore, the workshop data must be multiplied by 4.0 to conform with the present data.

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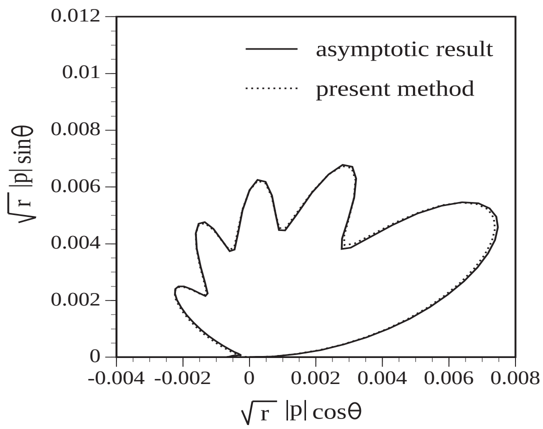


Figure 3: Directivity of unsteady pressure

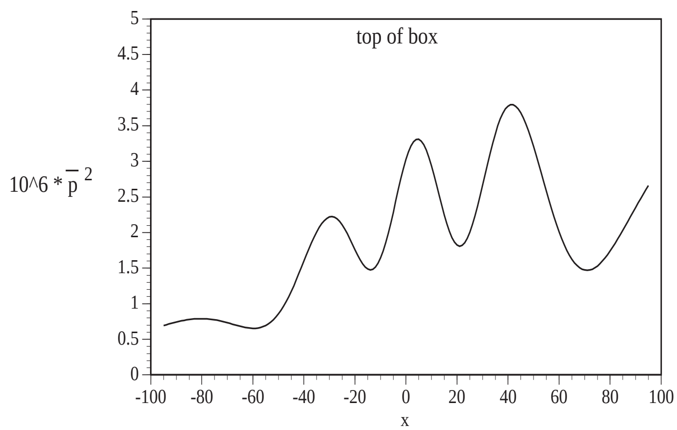


Figure 4: Unsteady pressure on top of the box for t = 0

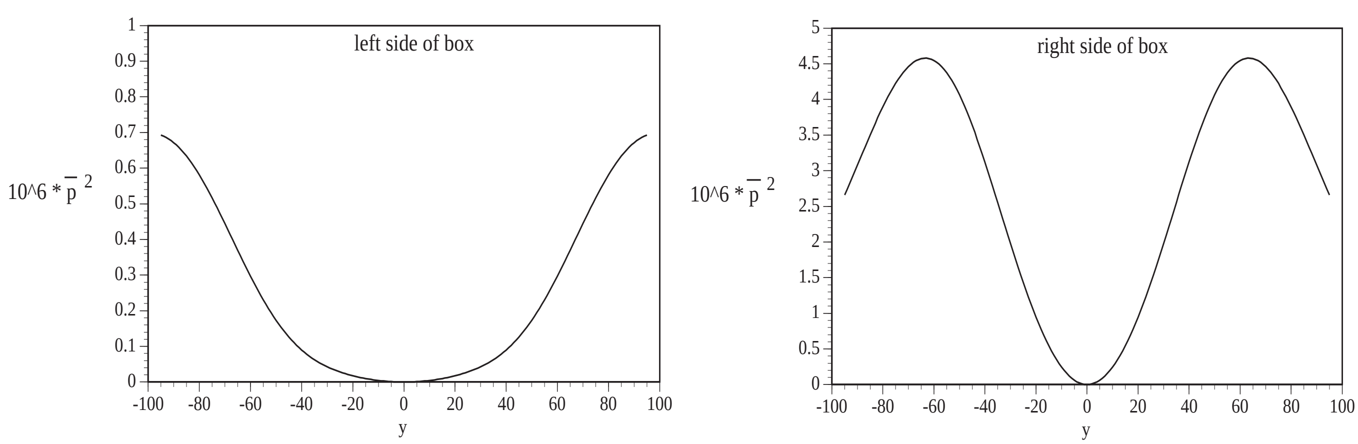


Figure 5: Unsteady pressure on left and right sides of the box for t = 0

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